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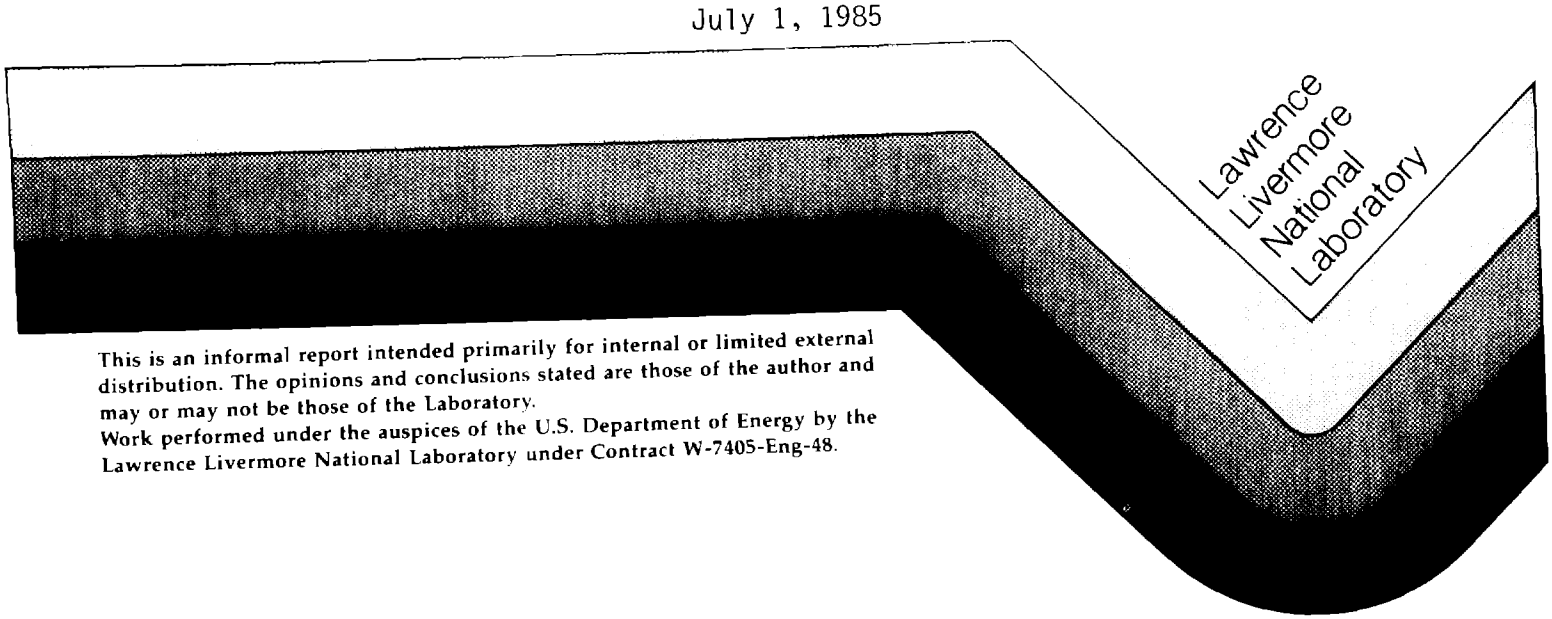
CONVERSION OF THE BOUNCE-AVERAGED FOKKER-PLANCK CODE TO CONSERVATION FORM

T. D. Rognlien

July 1, 1985



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I. INTRODUCTION

This report describes a major modification to the bounce-averaged Fokker-Planck code of Cutler et al.[1] The new version of the code is written in conservation form which results in the line density being conserved exactly except at phase space boundaries. The notation and procedure for writing the code in conservation form closely follows the work of Mirin on the square well code Hybrid II,[2] and Kerbel and McCoy on their bounce-average code CQL.[3]

Much of the original code has been preserved, and the input is the same as before; old input files should run on the new version. The major modifications to the code have occurred in subroutines COEF, IADVANCE, and RFTERMS. A new subroutine, FLUX, has been added to compute the flux across loss boundaries. The conservation code is called BACON11 with the last two numbers denoting the version. It can be obtained from FILEM on the MFECC computer system by typing

```
FILEM 356 RDS .BAFP BACON11 SLIB UPDATEC
```

The latter two library files are used in compiling and executing the code.

The motivation for rewriting the code was suspicious behavior of the original code for some problems. Perhaps most dramatic of these was the density runaway documented and studied by LoDestro.[4] She found that for a confining potential and a passing Maxwellian boundary condition, the code would not evolve to an isotropic Maxwellian, but rather the density would increase to a large value, producing a very non-Maxwellian distribution. The new version of the code does yield the expected isotropic Maxwellian.

The conservation version of the code is closely related to the finite difference code CQL written by Kerbel and McCoy.[3] Both are written in conservation form in v,θ coordinates. Some of the differences are as follows: BACON11 allows for spatially varying potentials, and CQL does not. BACON11 uses a 9-point operator and a fully implied ICCG solution; CQL does a more accurate differencing along separatrices which leads to more than a 9-point operator, and a splitting scheme is used for solving the matrix equation. BACON11 has a simple rf diffusion model, and CQL's rf model is

quite sophisticated. Another code which solves the same types of problems is the SMOKE code of Matsuda and Stewart.[5] This is a finite element code which allows multiple phase space regions and includes relativistic effects.

II. FOKKER PLANCK EQUATION

A. Local Fokker-Planck Equation

The local Fokker-Planck equation for species "a" can be written as

$$\frac{\partial f_a}{\partial t} + v \cos\theta \frac{\partial f_a}{\partial s} = \frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2\, \hat{e}_v \cdot \hat{\Gamma}_a\right) + \frac{1}{v^2 \sin\theta}\frac{\partial}{\partial \theta}\left(v \sin\theta\, \hat{e}_\theta \cdot \hat{\Gamma}_a\right) + S \quad (1)$$

where v, θ are local spherical coordinate velocity variables at the axial position s, and S is a source term. In this section, only the collisional contribution to $\hat{\Gamma}_a$ is considered; Sec. IV discusses the inclusion of rf. Following the notation of Mirin,[2] the collisional flux can be written as

$$\Gamma_a = \frac{1}{v^2}\left(A_a f_a + B_a \frac{\partial f_a}{\partial v} + C_a \frac{\partial f_a}{\partial \theta}\right)\hat{e}_v + \frac{1}{v \sin\theta}\left(D_a f_a + E_a \frac{\partial f_a}{\partial v} + F_a \frac{\partial f_a}{\partial \theta}\right)\hat{e}_\theta \quad (2)$$

The coefficients A_a - F_a are given in terms of Rosenbluth potentials G_a and I_a for particle species a

$$A_a = - v^2 \frac{\partial I_a}{\partial v} \quad , \quad (3)$$

$$B_a = \frac{v^2}{2}\frac{\partial^2 G_a}{\partial v^2} \quad , \quad (4)$$

$$C_a = - \frac{1}{2v}\frac{\partial G_a}{\partial \theta} + \frac{1}{2}\frac{\partial^2 G_a}{\partial v \partial \theta} \quad (5)$$

$$D_a = - \sin\theta \frac{\partial I_a}{\partial \theta} \quad (6)$$

$$E_a = \sin\theta\, C_a \quad (7)$$

$$F_a = \frac{\sin\theta}{2v^2}\frac{\partial^2 G_a}{\partial\theta^2} + \frac{\sin\theta}{2v}\frac{\partial G_a}{\partial v} \tag{8}$$

The Rosenbluth potentials can be written as

$$G_a = \sum_b \frac{4\pi e^4 Z_a^2 Z_b^2 \ln \Lambda_{ab}}{m_a^2} g_b \tag{9}$$

and

$$I_a = \sum_b \frac{4\pi e^4 Z_a^2 Z_b^2 \ln \Lambda_{ab}}{m_a m_b} h_b \quad . \tag{10}$$

Here the sum over b includes all charged species present. The actual potentials g_b and h_b are solutions to

$$\nabla^4 g_b = -8\pi f_b \quad , \tag{11}$$

and

$$\nabla^2 h_b = -4\pi f_b \quad , \tag{12}$$

where f_b is the distribution function of species b. The species density is $n_b = \int f_b \, d^3v$.

Note that Refs. 1 and 2 use the potential H_a instead of I_a which is related to I_a and G_a by $\nabla^2 G_a = 2(H_a - I_a)$. The coefficients A_a and D_a can be written more simply using I_a, and only first order derivatives of I_a are required. The different expressions of A_a and D_a obtained using I_a or H_a are analytically equivalent and numerically equivalent if the Legendre expansion is used to solve for g_b and h_b as is done here. However, if other numerical schemes are used to solve for g_b and h_b, the use of I_b could be numerically more accurate.

The code is currently set up to treat the interaction between one ion species and one electron species, and the distribution of only one of these species is actually computed. If the species being solved for is ions, the electrons are assumed to have an isotropic Maxwellian distribution. In this

case g_e and h_e are independent of pitch angle θ. If the species being solved for is electrons, the ions are assumed to be infinitely massive, causing only pitch scattering of the electrons. Then, the only ion potential which contributes is $g_i = n_i v$, n_i being the ion density. This term in turn only influences the pitch angle coefficient F_e.

The Rosenbluth potentials for the computed species are obtained by expanding the distribution function in Legendre functions. This is described in detail in Ref. 1 and 2, and the reader is referred there for further information.

B. Bounce-Averaged Fokker-Planck Equation

Using the assumption that collisions only weakly influence the orbit of a particle over its axial bounce orbit, Eq. (1) can be bounce averaged giving the evolution of the midplane distribution function

$$\frac{\partial f_a(v_0,\theta_0)}{\partial t} = \frac{1}{\tau_B}\oint \frac{ds}{v\cos\theta}\left[\frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2\hat{e}_v\cdot\hat{\Gamma}_a\right) + \frac{1}{v^2\sin\theta}\frac{\partial}{\partial\theta}\left(v\sin\theta\,\hat{e}_\theta\cdot\hat{\Gamma}_a\right)\right] \tag{13}$$

where (v_0, θ_0) are midplane coordinates. The source term S has been omitted since it is unchanged from that of Cutler et al.[1] The bounce time τ_B is

$$\tau_B = \oint \frac{ds}{v\cos\theta} \quad . \tag{14}$$

The particle orbits are defined by the equations for conservation of energy and magnetic moment:

$$v^2 = v_0^2 + v_p^2 \quad , \tag{15}$$

and

$$v^2\sin^2\theta = \psi\, v_0^2\sin^2\theta_0 \quad . \tag{16}$$

Here $v_p^2 = 2Z_a e\phi(s)/m_a$ and $\psi = B(s)/B_0$; ϕ is the electrostatic potential and B is the magnetic field. In converting Eq. (13) into a useful form in midplane variables, we need the transformations

$$\frac{\partial}{\partial v} = \frac{v}{v_0}\frac{\partial}{\partial v_0} - \frac{v_p^2}{v v_0^2}\tan\theta_0\frac{\partial}{\partial\theta_0} \tag{17}$$

and

$$\frac{\partial}{\partial\theta} = \frac{\tan\theta_0}{\tan\theta}\frac{\partial}{\partial\theta_0} \tag{18}$$

Using Eqs. (15-18), Eq. (13) can be transformed into

$$\frac{\partial f_a}{\partial t} = \frac{1}{\tau_B\, v_0 \cos\theta_0}\left[\frac{1}{v_0^2}\frac{\partial}{\partial v_0}\left(\bar{A}_a + \bar{B}_a\frac{\partial}{\partial v_0} + \bar{C}_a\frac{\partial}{\partial\theta_0}\right)f_a\right.$$

$$\left. + \frac{1}{v_0^2\sin\theta_0}\frac{\partial}{\partial\theta_0}\left(\bar{D}_a + \bar{E}_a\frac{\partial}{\partial v_0} + \bar{F}_a\frac{\partial}{\partial\theta_0}\right)f_a\right] \tag{19}$$

where the bars denote the bounce-averaged coefficients whose units differ from the local coefficients by a length dimension:

$$\bar{A}_a = v_0^2\cos\theta_0\oint\frac{ds}{v\cos\theta}\frac{A_a}{v} \tag{20}$$

$$\bar{B}_a = v_0\cos\theta_0\oint\frac{ds}{v\cos\theta}B_a \tag{21}$$

$$\bar{C}_a = v_0^2\cos\theta_0\oint\frac{ds}{v\cos\theta}\left(\frac{C_a}{v}\frac{\partial\theta_0}{\partial\theta} + \frac{B_a}{v}\frac{\partial\theta_0}{\partial v}\right) \tag{22}$$

$$\bar{D}_a = \oint\frac{ds}{v\cos\theta}v_0\left(\frac{\cos\theta}{\psi}D_a - \frac{v_p^2}{v^3}\sin^2\theta_0\,A_a\right) \tag{23}$$

$$\bar{E}_a = \oint\frac{ds}{v\cos\theta}\left(\frac{v}{\psi}\cos\theta\,E_a - \frac{v_p^2}{v^2}\sin^2\theta_0\,B_a\right) \tag{24}$$

$$\overline{F}_a = \oint \frac{ds}{v \cos\theta} v_0 \left[\frac{\cos\theta}{\psi} \left(F_a \frac{\partial \theta_0}{\partial \theta} + E_a \frac{\partial \theta_0}{\partial v} \right) \right.$$

$$\left. - \frac{v_p^2}{v^3} \sin^2 \theta_0 \left(C_a \frac{\partial \theta_0}{\partial \theta} + B_a \frac{\partial \theta_0}{\partial v} \right) \right] \qquad (25)$$

The required derivatives are obtained from Eqs. (17) and (18). Note that Eq. (19) is in the desired conservation form for the line density, N_L, given by

$$N_L = \int \tau_B v_0 \cos\theta_0 f_a \, d^3 v_0 \quad . \qquad (26)$$

The numerical procedure used to evaluate the bounce-average coefficients is described in Ref. 1. For $v_p = 0$, these coefficients reduce to those in Ref. 3.

III. NUMERICAL PROCEDURE

A. Finite Difference Equation

Equation (19) is written in finite difference form on a (θ_0, v_0) mesh with the indices (i, j) as described in Ref. 2 except that an implicit scheme is used, i.e., the RHS is evaluated at the advanced time. Specifically

$$\frac{f_{i,j}^{n+1} - f_{i,j}^{n}}{\Delta t} = \frac{1}{\tau_B v_j^3 \cos\theta_i} \left[\frac{1}{\Delta v_j} \left(J_{i,j+1/2}^{n+1} - J_{i,j-1/2}^{n+1} \right) \right.$$

$$\left. + \frac{1}{\Delta\theta_i \sin\theta_i} \left(K_{i+1/2,j}^{n+1} - K_{i-1/2,j}^{n+1} \right) \right] \qquad (27)$$

where the superscript denotes the time level, i.e., $t^{n+1} = t^n + \Delta t$. The grids run from $j = 2$ to $j = j_m$ and $i = 2$ to $i = i_m$ with i_m corresponding to $\theta = \pi/2 - \delta$. A "ghost" grid point is placed at $\theta = \pi/2 + \delta$ to aid in implementing the $\pi/2$ boundary condition; $\delta = \Delta\theta_{im}/2$. Unlike in the original code, the $\pi/2$ grid point is not used. The grid quantities are

$$\Delta v_{j\pm1/2} = \pm(v_{j\pm1} - v_j), \quad \Delta v_j = 0.5 \times (\Delta v_{j+1/2} + \Delta v_{j-1/2}) \quad ,$$

and θ quantities are defined similarly. The J's are

$$J_{i,j\pm1/2} = \frac{\overline{A}_{i,j}\, f_{i,j} + \overline{A}_{i,j\pm1}\, f_{i,j\pm1}}{2} \pm \frac{\overline{B}_{i,j\pm1/2}}{\Delta v_{j\pm1/2}} (f_{i,j\pm1} - f_{i,j})$$

$$+ \frac{\overline{C}_{i,j\pm1}}{4\Delta\theta_i} (f_{i+1,j\pm1} - f_{i-1,j\pm1}) + \frac{\overline{C}_{i,j}}{4\Delta\theta_i} (f_{i+1,j} - f_{i-1,j}) \qquad (28)$$

where the diffusion coefficients (i.e., $\overline{A}$ etc.) are obtained using the current value of $f = f^n$ in Eqs. (11-12), but the explicit f's are evaluated at the advanced time step, i.e., $f \to f^{n+1}$. Also,

$$\overline{B}_{i,j\pm1/2} = 0.5 \times (\overline{B}_{i,j} + \overline{B}_{i,j\pm1}) \quad .$$

Similarly,

$$K_{i\pm1/2} = \frac{\overline{D}_{i,j}\, f_{i,j} + \overline{D}_{i\pm1,j}\, f_{i\pm1,j}}{2} \pm \frac{\overline{F}_{i\pm1/2,j}}{\Delta\theta_{i\pm1/2}} (f_{i\pm1,j} - f_{i,j})$$

$$+ \frac{\overline{E}_{i\pm1,j}}{4\Delta v_j} (f_{i\pm1,j+1} - f_{i\pm1,j-1}) + \frac{\overline{E}_{i,j}}{4\Delta v_j} (f_{i,j+1} - f_{i,j-1}) \quad . \qquad (29)$$

The original version of the bounce-averaged code[1] used a splitting scheme as described in Ref. 2 for solving for $f^{n+1}_{i,j}$. Subsequently, Cutler implemented a fully implicit 9-point difference operator which was solved by Incomplete Cholesky Conjugate Gradient (ICCG) method. Although we now have a different form for the bounce-averaged Fokker-Planck equation, the same 9-point ICCG algorithm developed by Cutler can be used. The current version of the code uses a similar ICCG algorithm developed by Shestakov and Anderson.[6]

The 9-point operator form of Eq. (27) is

$$\frac{f^{n+1}_{i,j} - f^n_{i,j}}{\Delta t} = \sum_{\ell=-1}^{1} \sum_{m=-1}^{1} a(i, j, \ell, m)\, f^{n+1}_{i+\ell,j+m} \quad . \qquad (30)$$

The nine coefficients are

$$a(i,\ j,\ 0,\ 0) = -r\left(\frac{\overline{B}_{i,j+1/2}}{\Delta v_j\ \Delta v_{j+1/2}} + \frac{\overline{B}_{i,j-1/2}}{\Delta v_j\ \Delta v_{j-1/2}} + \frac{\overline{F}_{i+1/2,j}}{\sin\theta_i\ \Delta\theta_i\ \Delta\theta_{i+1/2}}\right.$$

$$\left. + \frac{\overline{F}_{i-1/2,j}}{\sin\theta_i\ \Delta\theta_i\ \Delta\theta_{i-1/2}}\right) , \quad (31)$$

$$a(i,\ j,\ \pm 1,\ 0) = \frac{r}{\Delta\theta_i\ \sin\theta_i}\left(\frac{\pm\overline{D}_{i\pm1,j}}{2} + \frac{\overline{F}_{i\pm1/2,j}}{\Delta\theta_{i\pm1/2}}\right) , \quad (32)$$

$$a(i,\ j,\ 0,\ \pm 1) = \frac{r}{\Delta v_j}\left(\pm\frac{\overline{A}_{i,j\pm1}}{2} + \frac{\overline{B}_{i,j\pm1/2}}{\Delta v_{j\pm1/2}}\right) , \quad (33)$$

$$a(i,\ j,\ \pm 1,\ -1) = \frac{\pm r}{4\ \Delta\theta_i\ \theta v_j}\left(\overline{C}_{i,j-1} + \frac{\overline{E}_{i\pm1,j}}{\sin\theta_i}\right) , \quad (34)$$

$$a(i,\ j,\ \pm 1,\ 1) = \frac{\pm r}{4\Delta\theta_i\ \Delta v_j}\left(\overline{C}_{i,j+1} + \frac{\overline{E}_{i\pm1,j}}{\sin\theta_i}\right) , \quad (35)$$

and the factor $r = 1/(\tau_B\ v_j^3 \cos\theta_i)$.

B. Boundary Conditions

The boundary conditions are implemented in much the same way as described by Mirin[2] with a few differences owing to the implicit scheme used here. The analytic boundary conditions are

$$f_a(v_{max},\theta) = 0 \quad (36)$$

$$\frac{\partial f_a(0,\theta)}{\partial v} = 0 \quad (37)$$

$$\frac{\partial f_a(v,0)}{\partial \theta} = 0 \tag{38}$$

and

$$\frac{\partial f_a(v,\pi/2)}{\partial \theta} = 0 \quad . \tag{39}$$

Also, along any loss boundary we have $f_a = 0$.

Numerically, Eqs. (37) to (39) are satisfied by imposing a zero flux condition across the boundaries. For Eq. (37) at $v = 0$, this yields

$$J^{n+1}_{i,3/2} = \frac{\overline{A}_{i,2} f^{n+1}_{i,2}}{2} + \frac{\overline{B}_{i,2}}{2} \frac{(f^{n+1}_{i,2} - f^{n+1}_{i,1})}{\Delta v_{3/2}}$$

$$+ \frac{\overline{C}_{i,2}}{4\Delta\theta_i} (f^{n+1}_{i+1,2} - f^{n+1}_{i-i,2}) = 0 \quad . \tag{40}$$

Because of the last term, we cannot use Eq. (40) directly and maintain the implicit 9-point difference scheme. By referring to Eq. (27), note that $J_{i,3/2} = 0$ can be satisfied approximately by setting the sum of the first two terms in Eq. (40) to zero and adding an explicit source-like term S_{bj}, to the RHS of Eq. (27) which cancels the third term in $J_{i,3/2}$. These two steps yield

$$f^{n+1}_{i,1} = f^{n+1}_{i,2} \left(1 + \frac{\overline{A}_{i,2}}{\overline{B}_{i,2}} \Delta v_{3/2}\right) \tag{41}$$

and

$$S_{bj} = \frac{\overline{C}_{i,2} (f^{n}_{i+1,2} - f^{n}_{i-1,2})}{4\tau_B(i,2)\ v_2^3\ \Delta v_2\ \Delta\theta_i \cos\theta_i} \quad . \tag{42}$$

Because of the explicit term S_{bj}, zero flux is not strictly maintained until steady state. If desired, this defect can be remedied by a more elaborate boundary condition. The effect which Eq. (41) has on the 9-point operator of Eq. (30) are the transformations

$$a(i,2,\ell,0) \rightarrow a(i,2,\ell,0) + a(i,2,\ell,-1)\ (1 - \bar{A}_{i+\ell,2}\ \Delta v_{3/2}/\bar{B}_{i+\ell,2})$$

and $a(i,2,\ell,-1) \rightarrow 0$.

In just the same manner, the boundary condition at i = 2 requires $K^{n+1}_{3/2,j} = 0$ which yields

$$f^{n+1}_{i,j} = f^{n+1}_{2,j}\left(1 + \frac{\bar{D}_{2,j}\ \Delta\theta_{3/2}}{\bar{F}_{2,j}}\right) \tag{43}$$

and an explicit source term

$$S_{bi} = \frac{\bar{E}_{2,j}(f^n_{2,j+1} - f^n_{2,j-1})}{4\tau_B(2,j)\ v_j^3\ \Delta v_j\ \Delta\theta_2\ \cos\theta_2\ \sin\theta_2} \quad . \tag{44}$$

Equation (43) is used to redefine a(2,j,0,m) and a(2,j,-1,m).

The boundary condition at $\theta = \pi/2$ is obtained by using a "ghost" grid point at $\theta = \pi/2 + \delta$ termed the i = I + 1 theta grid point; the i = I grid point has $\theta = \pi/2 - \delta$. Using symmetry about $\theta = \pi/2$ then gives f and the diffusion coefficients at the "ghost" grid point

$$f_{I+1,j} = f_{I,j}$$

$$\bar{D}_{I+1,j} = -\bar{D}_{I,j}$$

$$\bar{E}_{I+1,j} = -\bar{E}_{I,j} \quad .$$

These relations yield $K^{n+1}_{I+1/2,j} = 0$ and thus result in no flux crossing the $\theta = \pi/2$ boundary.

C. Computing the Line Flux

A useful diagnostic is the line flux across the loss boundary or across the boundary separating a fixed distribution from one which is evolved in

time. The line flux per unit velocity is obtained by integrating the RHS of Eq. (27) over the confined region of velocity space which leaves just the boundary terms

$$\Gamma_j = 8\pi \int_{\text{confined}} d\theta_0 \, \tau_B \, v_j^3 \cos\theta_0 \sin\theta_0 \, \text{RHS(27)}$$

$$= 8\pi \left(\mp \sum_i \Delta\theta_i \sin\theta_i \, J_{i,jby\pm 1/2} \Delta v_{jby} - K_{iby+1/2,j} \right) . \qquad (45)$$

Here jby is the value of j along the boundary in the θ direction, and iby is the value of i along the boundary in the v direction. One uses the upper (lower) sign in Eq. (45) if the confined region extends from v_{jby} (0) to ∞ (v_{jby}). A plot of Γ_j vs v_j shows the flux as a function of velocity. The total flux is

$$\Gamma_T = \sum_j \Gamma_j \Delta v_j \quad . \qquad (46)$$

The contributions to the flux from collisional and rf diffusion are separated and plotted on the same frame in the code output to enable comparison. The total flux aids in confirming that particle balance is maintained to a high degree of accuracy.

IV. RF CYCLOTRON HEATING

The code also includes diffusion due to rf cyclotron heating. We assume that the heating occurs in the velocity direction perpendicular to the magnetic field. Thus, a new term is added to the RHS of Eq. (1) of the form

$$\frac{\partial f_a}{\partial t} + v \cos\theta \frac{\partial f_a}{\partial s} = \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} v_\perp D_{rf} \frac{\partial f_a}{\partial v_\perp} + \text{collisional terms}$$

$$= \frac{1}{v^2} \frac{\partial}{\partial v} \left(B_{rf} \frac{\partial f_a}{\partial v} + C_{rf} \frac{\partial f_a}{\partial \theta} \right) + \frac{1}{v^2 \sin\theta} \frac{\partial}{\partial \theta} \left(E_{rf} \frac{\partial f_a}{\partial v} + F_{rf} \frac{\partial f_a}{\partial \theta} \right)$$

$$+ \text{collisional terms} \qquad (47)$$

where the rf diffusion terms are

$$B_{rf} = v^2 \sin^2\theta \, D_{rf}$$

$$C_{rf} = v \sin\theta \cos\theta \, D_{rf}$$

$$E_{rf} = v \sin^2\theta \cos\theta \, D_{rf}$$

$$F_{rf} = \sin\theta \cos^2\theta \, D_{rf} \tag{48}$$

The bounce-average coefficients are obtained by inserting the local coefficients into Eqs. (21), (22), (24), (25).

Up to this point, the form of D_{rf} has been unspecified. We use a model given by Rensink[7]

$$D_{rf} = D_0 \frac{B_r}{\Delta B_r \sqrt{\pi}} \exp\left(- [(B_r - B)/\Delta B_r]^2\right) \frac{x_\perp^{2\alpha}}{(1 + x_\perp^2)^\beta} \tag{49}$$

where B is the magnetic field, $x_\perp = v_\perp/v_n$, and α and β are specified by the input. Normally, ΔB_r is taken as small to localize the resonance spatially to simulate cyclotron heating. If D_{rf} has no explicit velocity dependence ($\alpha = \beta = 0$), it models fundamental cyclotron heating ($\omega = \omega_c$) for small perpendicular wavenumber $k_\perp v_t/\omega_c \ll 1$ where $v_t = \sqrt{2T/m}$ is the thermal velocity. For $\Delta B/B_r \ll 1$, there is a simple relation between D_0 and the power absorbed on one side of the midplane:

$$\text{Power absorbed} = 2nm \, D_0 \, L_B \, A \quad . \tag{50}$$

Here n is the local density, L_B the local magnetic scale length, A the area, and mD_0 (= EMDRF in the code) is the value of the diffusion coefficient in keV/s.

Heating at higher cyclotron harmonics can be modeled by choosing $\alpha = \ell - 1$, where ℓ is the harmonic number yielding the resonance $\omega = \ell\omega_c$. For $\ell = 2$ and $k_\perp v_t/\omega_c \ll 1$, the normalization velocity is $v_n = \sqrt{2}\,\omega/k_\perp$. The diagnostic in the code which gives the rf power out [Eq. (50)] is correct only for $\ell = 1$, but can be corrected for $\ell = 2$ and $k_\perp v_t/\omega_c \ll 1$ by multiplying the power by $(k_\perp v_t/2\omega_c)^2$.

V. SUMMARY AND ACKNOWLEDGMENTS

This report describes the conversion of the bounce-averaged Fokker-Planck code of Cutler et al.[1] to conservation form. The new code has been tested on several different problems with the SMOKE code[5] and agrees well. A new diagnostic has been added which allows calculation of the flux across loss boundaries. This is used to confirm that the line density is conserved to a high degree of accuracy--typically better than one part in 10^5. The effects of a time varying magnetic field, the so-called B terms,[1] have not been included in the conservation form as of this writing.

This project has benefitted by help from a number of people. Linda LoDestro and Marv Rensink have taken time to explain the details of the code. Bob Campbell has added finite transit losses and a new version[6] of the ICCG algorithm. Ray Jong and John Stewart have provided test case runs from SMOKE for comparison, and Art Mirin has tutored me on numerical conservation. Archer Futch pointed out an implausible result from an earlier version of this code which lead to the identification of a bug.

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